mum accuracy for azimuth lies along the plane that bisects the angle formed by the baselines; the maximum accuracy for elevation lies along the plane perpendicular to this bisector. Physically, the plane for maximum elevation accuracy occurs when

$$\partial \dot{E}/\partial \dot{p} = -(\partial \dot{E}/\partial \dot{q})$$

so that an error that occurs in $\dot{R}_0$ is cancelled when it appears in both $\dot{p}$ and $\dot{q}$. The same statement is true for azimuth also, of course.

Libration Damping and Lunar Mass

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GRAVITATIONALLY oriented satellites are of interest currently because of the possibility they afford to control orientation in space for long time durations without a continuing power requirement. A major design problem is the provision of adequate damping for rotational oscillations; the theory of physical librations of the moon is a useful guide to the underlying dynamics. Consideration of the interactions between librational and orbital motions leads to a new relationship between masses and orbital distances and reveals a condition on friction terms.

Energy dissipation within the moon is directly related to the couple-producing term in the gravitational potential, the sun's effect predominating over the earth's. If the energy of moon's orbital motion around earth is expressed as

$$k^2 M_e M_m / -2r$$

and its variation determined by dissipation, the relationship can be written as

$$\frac{M_e M_m}{r^2}\,\delta r + \frac{M_s M_m}{R^3}\delta\phi_1 = 0 \qquad (1)$$

where the Gaussian constant k^2 has been suppressed as a common factor. Earth, moon, and sun masses are denoted M_e, M_m, M_s; earth-moon and sun-moon mean distances are represented by r and R; effects of lunar orientation toward sun and lunar mass asymmetries determine ϕ_1, the explicit form of which is given by MacCullagh's formula (see, e.g., Ref. 1).

Tidal dissipation within the earth due to the proximity of the lunar mass can likewise be related to variation of earth's orbit around sun, giving

$$\frac{M_s M_e}{R^2}\,\delta R + \frac{M_e M_m}{r^3}\,\delta\varphi_2 = 0 \qquad (2)$$

where ϕ_2 likewise depends on the differences between earth's principal inertia moments and the earth-moon orientation.

Although the two Eqs. (1) and (2) do not form a simultaneous system, it is observed that the products of diagonal coefficients form an equality

$$(M_e M_m / M_s M_s) = (r/R)^5 \qquad (3)$$

as is easily verified by direct substitution of known numerical values for each of the quantities contained in this equation. It should be noted that Eq. (3) has not been deduced from (1) and (2), so that a further relationship is implied between the differential coefficients which they contain:

$$\frac{\delta\phi_1}{\delta\phi_2} = \frac{\delta R}{\delta r} \qquad (4)$$

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The ratio on the right side of (4) can be evaluated on the basis of orbital considerations (e.g., by requiring the total moment of momentum of earth and moon orbital motions to remain constant). Then the terms on the left relate dissipation within the satellite (moon) to that within the primary. Reciprocity relationships are suggested by the form of (4).

Equation (3) recalls the formula given by Laplace for the radius of the "activity sphere" outside of which the sun's attraction is more important than the earth's on the motion of a third body. If the geometric mean of earth and moon masses is replaced by earth mass, in fact, Eq. (3) becomes identical to Laplace's formula (see, e.g., Ref. 2). That relationship is known to be devoid of precise significance, despite its utility; in contrast, Eq. (3) may be considered surprising in its precision, despite the largely heuristic development given here.

The striking simplicity of the relationship (3), moreover, leads one to expect that it could be found in treatises on celestial mechanics; this is not the case. Nor does the classic literature appear to provide a sound dynamical basis for the equation.

References

[1] Routh, E. J., *Advanced Dynamics of Rigid Bodies* (Dover Publications, Inc., New York, 1955), p. 340.
[2] Tisserand, F., *Mécanique Céleste* (Gauthier-Villars, Paris, 1896), Tome IV, p. 200.

Shock Standoff Distance for Equilibrium Flow Around Hemispheres Obtained from Numerical Calculations

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KNOWLEDGE of the shock standoff distance yields valuable information concerning the flow field around blunt bodies including first-order estimates of shock-layer radiation and nonequilibrium flow effects. For a given set of freestream conditions and body shape, the standoff distance can be obtained from a numerical solution of the governing differential equations as is described, for example, in Refs. 1 and 2. However, in view of the wide variation in both the possible composition of planetary atmospheres and the possible vehicle trajectories, simple correlations would be useful for design purposes. It is shown in the present note that previously obtained correlations are applicable for a wide range of freestream conditions and gas mixtures.

The inverse method (wherein a shock shape is assumed and a body shape is calculated) has been used to obtain flow-field solutions for air, nitrogen, carbon dioxide, argon, and a mixture of 50% argon, 40% nitrogen, and 10% carbon dioxide. Details of the calculation method can be found in Ref. 1. The equilibrium thermodynamic properties of the gases considered were calculated by Harry Bailey of Ames Research Center. Solutions for hemispheres have been obtained for freestream densities of 10^{-1}, 10^{-3}, and 10^{-5} times earth sea-level density and for velocities in 10,000 fps increments up to approximately 60,000 fps, the upper limit being governed by the maximum temperature of 45,000°R.

The shock standoff distance obtained by the present numerical method is shown in Fig. 1 as a function of the

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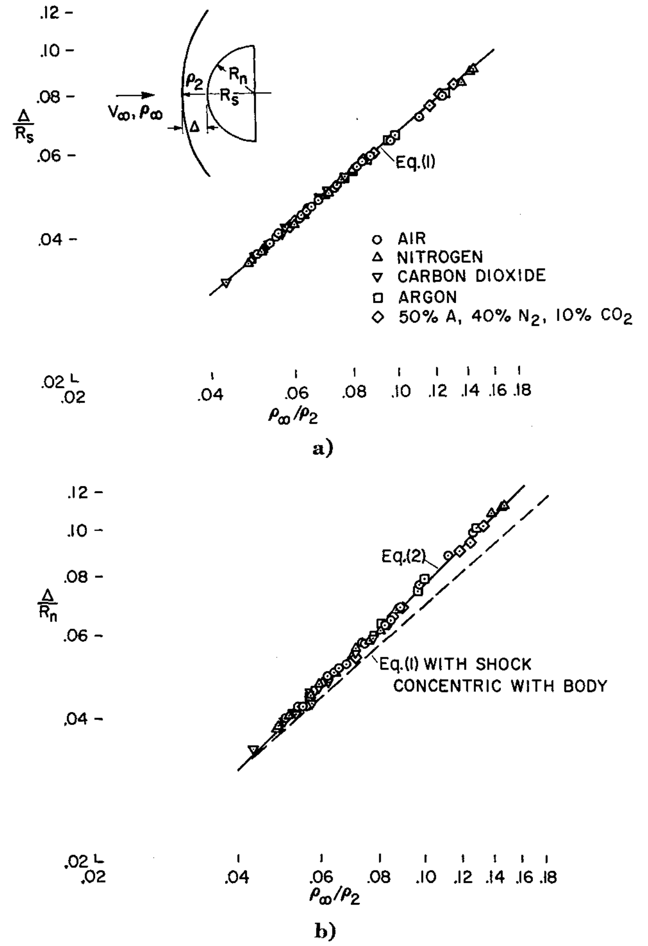

Fig. 1 Shock standoff distance as a function of density ratio across normal shock: a) Δ/R_s and b) Δ/R_n.

density ratio across a normal shock. In Fig. 1a the standoff distance Δ is normalized by R_s, the radius of curvature of the shock at the axis of symmetry. In this form the numerical results are correlated by the constant-density solution of Hayes and Probstein[3]:

$$\frac{\Delta}{R_s} = \frac{\rho_\infty/\rho_2}{1 + (8\rho_\infty/3\rho_2)^{1/2}} \tag{1}$$

Of more interest to the designer is the shock standoff distance normalized by R_n, the nose radius (Fig. 1b). The constant-density analysis by Hayes and Probstein does not yield the nose radius; hence, it is necessary to make a further

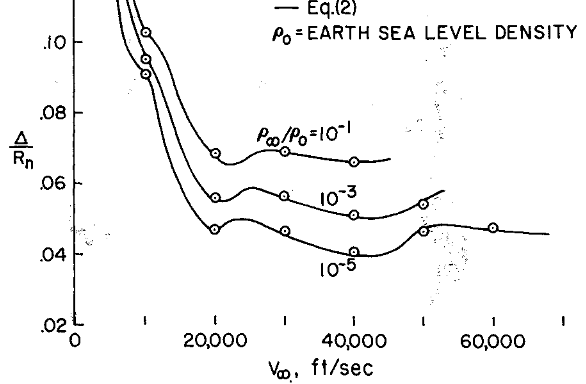

Fig. 2 Shock standoff distance for mixture of 50% argon, 40% nitrogen, and 10% carbon dioxide; T_∞ = 634° R.

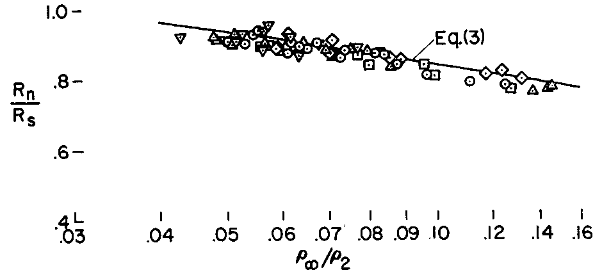

Fig. 3 Ratio of nose radius to shock radius.

assumption, for example, that the shock and body are concentric, in order to relate the standoff distance to the nose radius. This assumption is not valid as shown in Fig. 1b. However, the present numerical results are correlated by a simple linear law found by Seiff[4]:

$$\Delta/R_n = 0.78(\rho_\infty/\rho_2) \tag{2}$$

A typical variation of standoff distance with freestream velocity and density is shown in Fig. 2 for the mixture of argon, nitrogen, and carbon dioxide.

The ratio of nose radius to shock radius from the present numerical method is shown in Fig. 3. Combination of Eqs. (1) and (2) yields the following correlation equation:

$$\frac{R_n}{R_s} = \frac{1.28}{1 + (8\rho_\infty/3\rho_2)^{1/2}} \tag{3}$$

References

[1] Lomax, H. and Inouye, M., "Numerical analysis of flow properties about blunt bodies moving at supersonic speed in an equilibrium gas," NASA TR R-204 (1964).

[2] Klaimon, J. H., "Hypersonic flow field around a hemisphere in a CO_2-N_2-A gas mixture," AIAA J. **2**, 953–954 (1964).

[3] Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory* (Academic Press Inc., New York, 1959), pp. 158–162.

[4] Seiff, A., "Recent information on hypersonic flow fields," *Gasdynamics in Space Exploration*, NASA SP-24 (1962).

Heat Transfer from an Impinging Rocket Jet

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Nomenclature

A = area, ft^2
C_p = specific heat, Btu/lb-°F
h = heat-transfer coefficient, Btu/ft^2-°F-sec
k = thermal conductivity, Btu/ft-sec-°F
l = length into plate, ft
Pr = Prandtl number
q = heat flow rate, Btu/sec
t = time, sec
M = average molecular weight
T = temperature, °F
U = velocity, fps
W = weight, lb
X = distance from start of boundary layer, ft
γ = specific heat ratio
ρ = density, lb/ft^3
μ = viscosity, lb/in.-sec

Subscripts

g = gas
0 = stagnation
P = plate
R = reference

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